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Observing Dark Matter via the Gyromagnetic Faraday Effect

Susan Gardner
Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055

If dark matter consists of cold, neutral particles with a non-zero magnetic moment, then, in the presence of an external magnetic field, a measurable gyromagnetic Faraday effect becomes possible. This enables direct constraints on the nature and distribution of such dark matter through detailed measurements of the polarization and temperature of the cosmic microwave background radiation.

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Introduction. The existence of dark matter was first inferred in 1933 from Zwicky's observations of extragalactic nebulae [1]. In recent years, our ability to assay its abundance has sharpened considerably, and a concordance of disparate observations reveal that dark matter comprises some twenty-three percent of the energy density of the universe, with a precision of a few percent [2]. Yet, despite this progress, the fundamental nature of dark matter remains unclear. One cannot say with surety whether dark matter consists of a single species of particle, or of many, or even if it consists of stable, elementary particles at all. Dark matter could comprise aggregates of some kind, or be mimicked, in part, by a modification of gravity at large distances [3, 4, 5]. We do know that light, massive neutrinos cannot explain the galactic rotation curves [6], so that non-Standard-Model particles, arguably of the Fermi scale, are commonly invoked to explain it [7]. Accordingly, little, if anything, is known of each species' quantum numbers, mass, or mass distribution. In this Letter we consider the possibility that dark matter consists of neutral objects, which need not be elementary particles, of mass M with non-zero magnetic moments. The empirical limits on this possibility are weak and vary with the particle's mass [8].

Although our scenario naturally permits the dark constituents to be mutually interacting, as observational evidence suggests [9], it does differ significantly from usual ideas. For example, models of electroweak symmetry breaking with an additional discrete symmetry can yield viable dark matter candidates. In models with supersymmetry, the dark matter candidate — the "lightest supersymmetric particle" — is a Majorana particle, and its static magnetic moment is identically zero. Thus if the effect we discuss is observed, it demonstrates that supersymmetry does not provide an exclusive solution to the dark matter problem. On the other hand, models with "large" extra dimensions, such that their compactification radius R has $R^{-1} \lesssim 1$ TeV, offer dark matter candidates which are nominally consistent with our scenario [10]. In particular, models with universal extra dimensions [11] yield dark-matter candidates which are known to be compatible with observed constraints and which could also possess magnetic moments [12, 13, 14].

Let us now consider how cold dark matter with a non-zero magnetic moment can be observed. A medium of particles with either electric charges or magnetic moments develops a circular birefringence when subjected to an external magnetic field, even if the medium is isotropic. Consequently, the propagation speed of light in the medium will depend on the state of its circular polarization, so that light prepared in a state of linear polarization will suffer a rotation of the plane of that polarization upon transmission through the medium. If we define $k_\pm$ to be the wave number for states with right- $(+)$ or left-handed $(-)$ circular polarization, then the rotation angle is given by $\phi = (k_+ - k_-)l/2$, where l is the length of transmission through the medium. If the medium contains free electric charges, this is the Faraday effect known for light travelling through the electrons and magnetic fields of the warm interstellar medium (ISM) [15]. A Faraday effect can also occur in a magnetizable medium which is electrically neutral. The latter was first studied by Polder in a ferromagnetic medium [16, 17]. We term these the gyroelectric and gyromagnetic Faraday effects [18], respectively. We study the gyromagnetic Faraday effect associated with cold dark matter carrying a non-zero magnetic moment, though matter with a non-zero electric dipole moment and no magnetic moment could also generate an effect. We begin by comparing the Faraday effects in the ISM, for which the gyroelectric effect is familiar, before turning to a discussion of their impact on the cosmic-microwave background (CMB) polarization and the constraints such measurements can yield on models of dark matter.

Faraday Effects in the ISM. The ISM contains free electrons and external magnetic fields; it is gyroelectric and gives rise to a Faraday effect. We consider an external magnetic field $\mathbf{H}_0$ in the $\hat{\mathbf{z}}$-direction with circularly polarized electromagnetic waves propagating parallel to it. In this case, an electron with charge $-e$ and mass m suffers a displacement $\mathbf{s}$ via the Lorentz force

$$m\ddot{\mathbf{s}} = -e(\mathbf{E} + \dot{\mathbf{s}} \times \mathbf{H}_{\rm tot})\,, \tag{1}$$

where $\mathbf{H}_{\rm tot} = \mathbf{H}_0 + \mathbf{H}$. The electric field, e.g., associated with the wave is $\mathbf{E}(\mathbf{x},t) = E_\pm \mathbf{e}_\pm \exp(ik_\pm z - i\omega t)$, where $\mathbf{e}_\pm \equiv \hat{\mathbf{x}} \pm i\hat{\mathbf{y}}$. We define the polarization state with positive helicity, $\mathbf{e}_+$, to be right-handed, which differs from the convention used in optics. Assuming $|\mathbf{H}_0| \gg |\mathbf{H}|$, the steady-state solution for $\mathbf{s}$ yields the polarization $\mathbf{P} = e\mathbf{s}$ and the electric susceptibility χ_e, recalling $\mathbf{P}_\pm = \epsilon_0 \chi_{e\,\pm} \mathbf{E}_\pm$. We thus determine the permittivity $\epsilon_\pm$:

$$\frac{\epsilon_\pm}{\epsilon_0} \equiv 1 + \chi_{e\,\pm} = 1 - \frac{\omega_P^2}{\omega(\omega \mp \omega_H)} , \tag{2}$$

where the plasma frequency ω_P is given by $\omega_P^2 \equiv n_e e^2/\epsilon_0 m$, n_e is the electron number density, and $\omega_H = eH_0/m$. With $k_\pm = (\omega/c)\sqrt{\epsilon_\pm/\epsilon_0}$ and with $\omega \gg \omega_H, \omega_P$, we have $\phi = -\omega_P^2\omega_H l/2c\omega^2$ to leading order in ω. Generalizing this to variable electron densities and magnetic fields along the line of sight yields

$$\phi = -\frac{e^3}{2c\omega^2\epsilon_0 m^2}\int_0^l dz\, n_e(z) H_0(z) , \tag{3}$$

where $z = 0$ marks the location of the source. The ω dependence makes knowledge of the intrinsic source polarization unnecessary; one measures the position angle of linear polarization, in a fixed reference frame, as a function of ω, so that the line integral of $n_e(z)H_0(z)$ can be inferred [19, 20]. A pulsed radio source also permits the measurement of the frequency dependence of the arrival time, or time delay, which yields the line integral of $n_e(z)$ [19], so that the average magnetic field along the line of sight can also be determined.

The electrons' magnetic moments can be aligned to generate a net magnetization in a magnetic field at low temperature, so that the ISM can be regarded as gyromagnetic as well. We shall treat the gyroelectric and gyromagnetic effects independently. Applying a magnetic field in a gyromagnetic medium, namely, induces a magnetization $\mathbf{M}_{\rm tot}$, i.e., a net magnetic moment/volume, where $\mathbf{M}_{\rm tot} = \hat{\mathbf{z}} M_0 + \mathbf{M}$ and M_0 results from H_0 alone. The resulting magnetization obeys

$$\dot{\mathbf{M}}_{\rm tot} = \gamma \mathbf{M}_{\rm tot} \times \mathbf{H}_{\rm tot} , \tag{4}$$

where γ is the gyromagnetic ratio of the magnetic-moment-carrying particle. If the medium's constituents possess an electric dipole moment as well, an additional term appears in the Larmor formula [21]. We assume $|\mathbf{H}_0| \gg |\mathbf{H}|$, $|\mathbf{M}_0| \gg |\mathbf{M}|$, and the conventions of the gyroelectric case to determine the steady-state solution, which, neglecting the $\mathbf{M}\times\mathbf{H}$ term, is

$$M_\pm = \pm\frac{\chi_0\omega_H}{\omega \pm \omega_H} H_\pm \equiv \chi_\pm H_\pm , \tag{5}$$

where $\chi_0 \equiv M_0/H_0$ and $\omega_H \equiv \gamma H_0$. We recall the magnetic susceptibility χ_m is $\mathbf{M} = \chi_m \mathbf{H}$, so that

$$\frac{\mu_\pm}{\mu_0} \equiv 1 + \chi_{m\,\pm} = 1 \pm \frac{\chi_0\omega_H}{\omega \pm \omega_H} , \tag{6}$$

where $k_\pm = (\omega/c)\sqrt{\mu_\pm/\mu_0}$. Noting $\omega_H/\omega \ll 1$ and working to leading order in this quantity, one has $k_{\rm avg} \equiv (k_+ + k_-)/2$, which enters the time delay, with

$$k_{\rm avg} = \frac{\omega}{c}\left(1 - \frac{1}{2}\chi_0\left(\frac{\omega_H}{\omega}\right)^2 - \frac{1}{8}\chi_0^2\left(\frac{\omega_H}{\omega}\right)^2 + \dots\right) , \tag{7}$$

and $k_{\rm diff} = k_+ - k_-$, which enters the Faraday rotation angle ϕ, with

$$k_{\rm diff} = \frac{\chi_0\omega_H}{c} + \frac{\chi_0\omega_H^3}{c\,\omega^2} + \frac{\chi_0^2\omega_H^3}{2c\,\omega^2} + \dots . \tag{8}$$

The magnetization induced by H_0 is

$$M_0 = n_e\mu\tanh\left(\frac{\mu H_0}{k_B T}\right) = n_e\left(\frac{\mu^2 H_0}{k_B T}\right) , \tag{9}$$

where the corrections to the last equality are trivially small in the ISM. Diverse environmental conditions do exist in the ISM, but the magnetic field H_0 is no larger than a few μG — and its cold patches are no colder than a few 100 K [19]. Consequently, we can neglect non-leading powers in χ_0 with impunity. Defining the arrival time τ as $\tau \equiv l/v_{\rm g}$, where $v_g = d\omega/dk_{\rm avg}$, we thus have

$$\tau_{\rm delay} \equiv \tau - \lim_{\omega\to\infty}\tau = \frac{\mu^2\gamma^2}{2c\,\omega^2 k_B}\int_0^l dz \frac{n_e(z)H_0^2(z)}{T(z)} , \tag{10}$$

where we recall $\gamma = g\mu/\hbar$ and g is the usual Landé factor. The time delay contains the same frequency dependence as the result familiar from the gyroelectric case [19], though the appearance of H_0^2 makes this contribution a much smaller one in the ISM. We separate the rotation angle ϕ into frequency-independent and frequency-dependent pieces, so that $\phi = \phi_0 + \phi_\omega$, to yield

$$\phi_0 = \frac{\mu^2\gamma}{2ck_B}\int_0^l dz \frac{n_e(z)H_0(z)}{T(z)} , \tag{11}$$

$$\phi_\omega = \frac{\mu^2\gamma^3}{2c\omega^2 k_B}\int_0^l dz \frac{n_e(z)H_0^3(z)}{T(z)} , \tag{12}$$

where $\phi_\omega \sim 1/\omega^2$ as in the gyroelectric case. Once again the appearance of an extra factor of H_0^2 makes this contribution a much smaller one in the ISM. Unique to this case, however, is a frequency-independent shift ϕ_0. Its presence impacts the source polarization inferred from conventional Faraday rotation experiments. If we assume the T variation is small along the line of sight then the frequency-independent gyromagnetic Faraday effect shares a common integral with the usual gyroelectric effect. In this case we can then compare

$$\phi_0 = \frac{\mu^2\gamma}{2ck_B T}\int_0^l dz\, n_e(z)H_0(z) \tag{13}$$

with

$$\phi = \frac{e^3}{2c\omega^2\epsilon_0 m^2}\int_0^l dz\, n_e(z)H_0(z) \tag{14}$$

by computing

$$\tilde{\chi} \equiv \frac{\gamma\mu^2}{k_B T} = \frac{g\mu^3}{\hbar k_B T} \sim \frac{2\mu_B^3}{\hbar k_B T} \sim 1.5\cdot 10^{-7}\left[\frac{300\,{\rm K}}{T}\right]\frac{{\rm cm}^3}{{\rm G\ s}}$$

and

$$\chi \equiv \frac{e^3}{\omega^2\epsilon_0 m^2} = \frac{\alpha}{\pi}\cdot\frac{\hbar}{mc}\cdot\frac{e}{m}\cdot\lambda^2 \sim 1.6\cdot 10^{-6}\left[\frac{\lambda}{1\,\mathrm{cm}}\right]^2\frac{\mathrm{cm}^3}{\mathrm{G\ s}}\,,$$

where $g \sim 2$, $\mu_B \sim 5.79\cdot 10^{-9}$ eV/G, $k_BT \sim 1/40$ eV for $T \sim 300$ K, $1\,\mathrm{eV} \sim 6.37\,\mathrm{G}^2\mathrm{cm}^3$, $\alpha \sim 1/137$, $e/m \sim 1.76\cdot 10^7$ rad/Gs, and $\hbar/mc \sim 3.86\cdot 10^{-11}$ cm. Recent surveys have used wavelengths in the $\lambda = 6$ and 20 cm bands [19, 22]. We note, too, that most Faraday rotation accrues in the warm ISM, for which $T \sim 5000$ K. These effects, taken in concert, make the gyromagnetic effect much less than 1% of the gyroelectric one for radio sources. Nevertheless, as measurements of the position angle can be made to $\mathcal{O}(1^\circ)$ precision [19, 22], the gyromagnetic effect is still readily appreciable. The frequency-dependent gyromagnetic rotation ϕ_ω is relatively trivial, however; it is smaller than ϕ_0 by a factor of $\gamma^2 H_0^2/\omega^2 \sim 9\cdot 10^{-21}[\lambda/(1\,\mathrm{cm})]^2$, using $H_0 \sim 10^{-6}$ G.

Faraday Effects on the CMB Polarization. Our study of the gyromagnetic Faraday effect suggests that the frequency-independent rotation ϕ_0 is numerically most important, though its very lack of frequency dependence means we must employ sources of known polarization to determine it. To realize this, we turn to the CMB radiation, for the scalar gravitational perturbations which dominate the temperature fluctuations in inflationary cosmologies give rise to E-mode, or gradient-type, polarization exclusively [23, 24]. The Faraday effects provide a mechanism by which B-mode, or curl-type, polarization can be produced from an initial state of E-mode polarization; ultimately, we wish to interpret the B-mode polarization as a constraint on dark matter with a magnetic and/or electric dipole moment. A variety of sources of B-mode polarization exist, however, and it is important to separate the possibilities. Let us enumerate some of them explicitly. Primordial tensor or vector gravitational perturbations in the CMB can give rise to B-mode polarization [23, 24], and B-mode polarization can be generated from primordial E-mode polarization via gravitational lensing of the CMB by matter [25]. Magnetic fields can also imprint B-mode polarization. Primordial magnetic fields can do this both through the perturbations they directly engender [26], as well as through the gyroelectric Faraday rotation they mediate given an initial E-mode polarization [27]. Large-scale magnetic fields in galactic clusters [22] can also give rise to gyroelectric Faraday rotation at a much later stage [29], impacting the CMB polarization at small angular scales [23, 24, 28]. The CMB polarization is a continuously varying function across the sky, so that Faraday rotation measurements, in concert with other measurements to reconstruct the electron density distribution, can be used to map the distribution of intercluster magnetic fields [29]. The gyroelectric Faraday effects are distinguished by their ω^{-2} frequency dependence; the B-polarizations engendered by gravitational lensing and radiation are frequency-independent.

The gyromagnetic Faraday effect we have discussed can operate in all the cases in which the gyroelectric Faraday effect has been suggested. Its dominant effect is frequency-independent, so that, unlike the gyroelectric case, it is not crisply separable from other sources of B-mode polarization. CMB polarization measurements are realized in the 30-500 GHz range, so that the gyromagnetic effect is relatively much larger than it was for radio frequencies. Indeed, for a frequency of 150 GHz, or $\lambda = 0.2$ cm, and $T = 5000$ K, the gyromagnetic effect for electrons is some 10% of the gyroelectric one. We note, in passing, that, unlike the gyroelectric case [27], the rotations ϕ_0 and ϕ_ω from primordial magnetic fields can evolve in time; in particular, $H \propto a^{-2}$, where a is the cosmological expansion parameter — so that the rotation angles can be larger in epochs when the medium is relativistic.

Constraining Dark Matter. We now consider how the gyromagnetic Faraday effect can be used to constrain models of dark matter. Let us work in the context of a specific scenario. That is, we assume that systematic measurements of the B-mode polarization at small angular scales have been made, and have been cleansed of possible gravitational lensing contamination [30], and, moreover, that the mapping of the intergalactic magnetic field of Ref. [29], though challenging, has been realized. The last need not be done in its entirety — to start, it would be sufficient to have knowledge of the magnetic field along the line of sight associated with a measured patch of B-mode polarization. The observational studies needed to realize the field map imply that we ought have some knowledge of the temperature of the intergalactic medium (IGM) associated with regions of significant magnetic field as well. If all this is so, then the gyromagnetic rotation ϕ_0 associated with electrons in the IGM should be known, perhaps even as a varying function across the sky. If we neglect primordial sources of B-mode polarization, this ϕ_0 is itself a B-mode polarization. A significant, non-zero difference between it and the observed B-mode polarization at small angular scales can be attributed to the gyromagnetic Faraday rotation due to cold dark matter, carrying a non-zero magnetic or electric dipole moment. We have neglected the role of gravitational radiation, and of other B-mode sources [31], at small angular scales. We note that gravitational waves, in particular, could still prove observable at larger angular scales.

We should ascertain whether it is plausible that dark matter could engender a measureable Faraday rotation. The dark constituents possess non-zero magnetic or electric dipole moments, which is unusual [7]. We have also asserted that the constituents are "cold" — this enters in two points of our analysis. The first is crucial: T must be small enough that the magnetization of Eq. (9)

is not negligibly small. We note, however, that although $\mu_B H_0/k_B T \ll 1$ for electrons in the ISM and the IGM, measureable Faraday rotations can nevertheless accrue. The second is the use of the Larmor precession formula, Eq. (4); this implicitly assumes that the particles are cold, so that their motion is non-relativistic, though the electrons of the warm ISM and IGM are non-relativistic nevertheless. We can remove this restriction by replacing Eq. (4) with its relativistic analogue, stemming from the Bargmann-Michel-Telegdi (BMT) equation [21], so that it is not a limitation. To compute ϕ_0 for cold dark matter, we use Eq. (11), replacing $\mu_B \to \mu_{DM}$ and $n_e(z) \to n_{DM}(z)$. We estimate n_{DM} by dividing the average mass density in dark matter [2] by the mass of its constituent M_{DM}, though n_{DM} can locally vary due to clumping from gravitational interactions. We expect $\mu_{DM} \propto M_{DM}^{-1}$, so that $\phi_0 \propto M_{DM}^{-4}$, where we note that a composite particle can have $g \sim \mathcal{O}(1)$. If M_{DM} is of MeV scale, as some observational evidence suggests [32], and we assume $\mu_{DM} \sim \mu_B$, then ϕ_0 should be appreciable in comparison to the Faraday rotations of Ref. [29].

Summary. A Faraday effect also exists for light transiting a medium of electrically neutral particles with non-zero magnetic moments in an external magnetic field. We have shown that dark matter can generate such a gyromagnetic Faraday effect and that this possibility serves as a new source of B-mode polarization in the CMB. It should be possible to disentangle this new source of B-mode polarization from other sources, so that a non-zero effect due to such dark matter can be found, if it exists, with the implication that supersymmetric models do not provide an exclusive solution to the dark matter problem. The gyromagnetic Faraday effect can thus be used to probe the nature and distribution of dark matter, to realize a picture of our Universe shaped by what we *observe*, rather than by what we believe to be so.

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